1.4: Extrema and Average Rates of Change

Increasing and Decreasing Behavior

An analysis of a function can also include a description of the intervals on which the function is increasing, decreasing or constant,

Key Concept: Increasing, Decreasing, and Constant Functions

| Words | A function \( f \) is increasing on an interval \( I \) if and only if for any two points in \( I \), a positive change in \( x \) results in a positive change in \( f(x) \). |
| Symbols | For every \( x_1 \) and \( x_2 \) in an interval \( I \), \( f(x_1) < f(x_2) \) when \( x_1 < x_2 \). |

Example:

\[
\begin{align*}
\text{Interval: } & (-\infty, \infty) \\
\end{align*}
\]

| Words | A function \( f \) is decreasing on an interval \( I \) if and only if for any two points in \( I \), a positive change in \( x \) results in a negative change in \( f(x) \). |
| Symbols | For every \( x_1 \) and \( x_2 \) in an interval \( I \), \( f(x_1) > f(x_2) \) when \( x_1 < x_2 \). |

Example:

\[
\begin{align*}
\text{Interval: } & (-\infty, \infty) \\
\end{align*}
\]

| Words | A function \( f \) is constant on an interval \( I \) if and only if for any two points in \( I \), a positive change in \( x \) results in a zero change in \( f(x) \). |
| Symbols | For every \( x_1 \) and \( x_2 \) in an interval \( I \), \( f(x_1) = f(x_2) \) when \( x_1 < x_2 \). |

Example:

\[
\begin{align*}
\text{Interval: } & (a, b) \\
\end{align*}
\]

Example 1: Analyze Increasing and Decreasing Behavior.

Use the graph of each function to estimate intervals to the nearest .5 units on which the function is increasing, decreasing, or constant. Support your answer numerically.

1.

![Graph of f(x) = 2x - 8x + 5](image)

2.

![Graph of f(x) = \begin{cases} 3x + 11 & \text{if } x < -3.1 \\ 1.7 & \text{if } x \geq -3.1 \end{cases}](image)
3. \( f(x) = x^2 - 4 \)

The points at which a function changes its increasing or decreasing behavior are seen as peaks or valleys on its graph. At these critical points, called extrema, the function has a maximum or a minimum value, either relative or absolute.

**Relative Minimum:**

**Relative Maximum:**

**Absolute Minimum:**

**Absolute Maximum:**

**Example 2: Estimate and Identify Extrema of a Function**

Estimate and classify the extrema for the graph of each function. Support the answers numerically.

4. \( f(x) = -x^4 - x^3 + 3x^2 + 2x \)

5. \( g(x) = x^4 - 2x^3 - 2x + 3x^2 \)
Example 3: Using your Calculator to Approximate Extrema

Approximate to the nearest hundredth the relative or absolute extrema of each function. State the x-values that occur.

6. \[ h(x) = 7 - 5x - 6x^2 \]

7. \[ g(x) = 2x^3 - 4x^2 - x + 5 \]

Example 4: Find Average Rates of Change

Find the average rate of change of each function on the given interval.

8. \[ f(x) = x^3 - 2x^2 - 3x + 2; \ [2, 3] \]

9. \[ f(x) = x^4 - 6x^2 + 4x; \ [-5, -3] \]